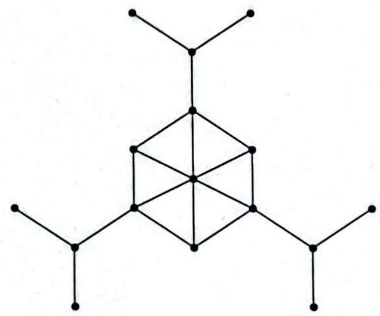
Exam Graph Theory

27 January 2026, 11:45–13.45

- It is not allowed to use calculators, phones, computers, books, notes, the help of others or any other aids.
- Make sure to state clearly any results from the lecture notes you are using.
- Write the answer to each question on a separate sheet, **with your name and student number on each sheet**. This is worth 10 points (out of a total of 100). Nota bene: by a 'sheet' we mean a folded booklet with the university logo on the front.

Exercise 1 (20 points)
Find a maximum matching of this graph and show that it is maximum.



Exercise 2 (25 points)
Let $G = (V, E)$ be a connected graph with distinct edge weights and let T be its minimum spanning tree. For $V = A \uplus B$ a partition of the vertices (with A and B non-empty), *the smallest* (A, B)*-edge* is the edge with one endpoint in A and one endpoint in B with the smallest weight.

Show that for $e \in E$, e is in T if and only if there is a partition $V = A \uplus B$ for which e is the smallest (A, B)-edge.

Exercise 3 (20 points)
Let $A_1, \dots, A_n$ be finite sets. Consider the following graph $G = (V, E)$. Let

$$V = A_1 \times \dots \times A_n = \{(a_1, \dots, a_n) : a_1 \in A_1, \dots, a_n \in A_n\}$$

and for $v, v' \in V$, we have that $vv' \in E$ if and only if v and v' differ in all coordinates.

Show that $\chi(G) = \min_{i=1,\dots,n} |A_i|$. *Hint: for the lower bound, look for a clique.*

Exercise 4 (25 points)
Let S be a set of $n \geq 3$ points in $\mathbb{R}^2$ such that for all distinct $x, y \in S$ the distance between them is at least 1.

(a) Prove that there are at most $3n - 6$ pairs of points from S that have distance exactly equal to 1.

(b) Without using a result from the lecture notes, show that there is a partition $S = S_1 \uplus S_2 \uplus \dots \uplus S_7$, so that, for $i = 1, 2, \dots, 7$, for any pair of vertices in S_i, the distance between them is larger than 1.

Bonus (+10 points): Without using a result from the lecture notes, show that there is a partition $S = S_1 \uplus S_2 \uplus \dots \uplus S_4$, so that, for $i = 1, 2, \dots, 4$, for any pair of vertices in S_i, the distance between them is larger than 1. *Hint for bonus: Consider a rotation of* $\mathbb{R}^2$ *such that all* x*-coordinates of points in* S *are distinct. Now decide on an order to assign the vertices to one of the sets.*

(The end)